

BRST symmetries in SU(3) linear sigma model

S.-T. Hong^{1,2}, S.H. Lee²

¹ Department of Physics and Basic Science Research Institute, Sogang University, C.P.O. Box 1142, Seoul 100-611, Korea

² Department of Physics and Institute of Physics and Applied Physics, Yonsei University, Seoul 120-749, Korea

Received: 16 November 2001 / Revised version: 23 February 2002 /
Published online: 26 July 2002 – © Springer-Verlag / Società Italiana di Fisica 2002

Abstract. We study the BRST symmetries in the SU(3) linear sigma model which is constructed through the introduction of a novel matrix for the Goldstone boson fields satisfying geometrical constraints embedded in a SU(2) subgroup. To treat these constraints we exploit the improved Dirac quantization scheme. We also discuss phenomenological aspects in the mean field approach to this model.

1 Introduction

There have recently been considerable discussions concerning the strangeness in hadron physics. The SAMPLE experiment [1] reported the proton's neutral weak magnetic form factor, which has been suggested by the neutral weak magnetic moment measurement through parity violating electron scattering [2]. In fact, the SAMPLE Collaboration obtained the positive experimental result for the proton strange magnetic form factor [1] $G_M^s(Q^2 = 0.1(\text{GeV}/c)^2) = +0.14 \pm 0.29 (\text{stat}) \pm 0.31 (\text{sys})$. This positive experimental value is contrary to the negative values of the proton strange form factor which results from most of the model calculations except the predictions [3] based on the SU(3) chiral bag model [4] and the recent predictions of the chiral quark soliton model [5] and the heavy baryon chiral perturbation theory [6]. (See [7] for more details.)

It is also well known in strangeness hadron physics that kaon condensation [8–11] in nuclear dense matter may have an impact on the formation of low mass black holes instead of neutron stars for masses on the order of 1.5 solar masses [12]. Beginning with the proposal of kaon condensation in 1986 [8], the theory of kaon condensation in dense matter has become one of the central issues in nuclear physics and astrophysics together with the supernova collapse. The K^- condensation at a few times nuclear matter density was later interpreted [9] in terms of cleaning of $\bar{q}q$ condensates from the quantum chromodynamics vacuum by dense nuclear matter and also in terms of phenomenological off-shell meson–nucleon interactions [13]. Recently, kaon condensation was revisited in the context of the color superconductivity in the color–flavor-locking phase [11]

On the other hand, the Dirac method [14] is a well-known formalism to quantize physical systems with constraints. In this method, the Poisson brackets in a second-class constraint system are converted into Dirac brackets to attain self-consistency. The Dirac quantization scheme

has also been applied to nuclear phenomenology [15,16]. The Dirac brackets, however, are generically field-dependent, being nonlocal, and they contain problems related to the ordering of field operators. These features are unfavorable for finding canonically conjugate pairs. However, if a first-class constraint system can be constructed, one can avoid introducing the Dirac brackets and can instead use Poisson brackets to arrive at the corresponding quantum commutators. To overcome the above problems, Batalin, Fradkin, and Tyutin [17] developed a method which converts the second-class constraints into first-class ones by introducing auxiliary fields. Recently, this improved Dirac formalism has been applied to several models of current interest. (See [7] for more details.) In particular, the relation between the standard Dirac and improved Dirac schemes was clarified in the SU(2) skyrmion model by introducing additional correction terms [18]. Recently, the improved Dirac Hamiltonian method was also applied to the SU(2) and SU(3) skyrmion models [19,20] to construct a first-class Hamiltonian and the BRST symmetries, and to the superqualiton model [21] to investigate the color superconductivity in color–flavor-locking phase. Moreover, the BRST symmetry [22] has been studied [19] in the SU(2) skyrmion in the framework of the BFV formalism [23] which is applicable to theories with first-class constraints by introducing canonical sets of ghosts and anti-ghosts together with auxiliary fields. The BRST symmetry can also be constructed by using the residual gauge symmetry interpretation of the BRST invariance [24].

The motivation of this paper is to systematically apply the improved Dirac scheme [17,7] to the SU(3) linear sigma model to construct the BRST symmetries in this phenomenological model. In Sect. 2 we construct the SU(3) linear sigma model by introducing a novel matrix for the Goldstone bosons which satisfy geometrical second-class constraints. To treat these constraints we exploit the improved Dirac scheme to convert the second-class constraints into first-class ones. In Sect. 3 we construct

first-class physical fields and directly derive a compact expression of a first-class Hamiltonian in terms of these fields. We construct in Sect. 4 the BRST invariant effective Lagrangian and its corresponding BRST transformation rules in the SU(3) linear sigma model. In Sect. 5 we discuss the phenomenology of the pion and kaon condensates.

2 Model and constraints

In this section we apply the improved Dirac scheme [17, 7] to the SU(3) linear sigma model, which is a second-class constraint system. We start with the SU(3) linear sigma model Lagrangian of the form

$$L = \int d^3x \left[\frac{1}{2} \text{tr}(\partial_\mu M \partial^\mu M^\dagger) - \frac{1}{2} \mu_0^2 \text{tr}(M M^\dagger) - \frac{1}{4} \lambda_0 [\text{tr}(M M^\dagger)]^2 + \bar{\psi} i \gamma^\mu \partial_\mu \psi - g_0 (\bar{\psi}_L M \psi_R + \bar{\psi}_R M^\dagger \psi_L) \right], \quad (2.1)$$

where we have introduced a novel matrix for the Goldstone bosons satisfying geometrical second-class constraints¹

$$M = \frac{1}{\sqrt{2}} (\sigma \lambda_0 + i \pi_a \lambda_a), \quad a = 1, \dots, 8, \quad (2.2)$$

with $\lambda_0 = (2/3)^{1/2} I$ (I is the identity) and the Gell-Mann matrices normalized to satisfy $\lambda_a \lambda_b = (2/3) \delta_{ab} + (i f_{abc} + d_{abc}) \lambda_c$. Here we have meson fields $\pi_a = (\pi_i, \pi_M, \pi_8)$ with π_i, π_M and π_8 being the pion, kaon and eta fields, respectively, and the chiral fields ψ_L and ψ_R defined as

$$\psi_{R,L} = \frac{1 \pm \gamma_5}{2} \psi. \quad (2.3)$$

Note that we have used the SU(3) linear sigma model with the U(3) \times U(3) group structure so that we could incorporate the σ field consistently, as in the chiral bag model [26].

The Lagrangian (2.1) can then be rewritten in terms of the meson fields π_a as follows:

$$L = \int d^3x \left[\frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi_a \partial^\mu \pi_a) - \frac{1}{2} \mu_0^2 (\sigma^2 + \pi_a \pi_a) - \frac{1}{4} \lambda_0 (\sigma^2 + \pi_a \pi_a)^2 + \bar{\psi} i \gamma^\mu \partial_\mu \psi - g_0 \bar{\psi} \frac{1}{\sqrt{2}} (\sigma + i \gamma_5 \pi_a \lambda_a) \psi \right], \quad (2.4)$$

where we have assumed the SU(3) flavor symmetry for simplicity. Here the sigma and pion fields (σ, π_i) are constrained to satisfy the geometric constraints on the SU(2) subgroup manifold

¹ In previous works [25], authors have used a different ansatz for M such as $M = \sum_{i=0}^8 (\sigma'_i + i \pi_i) \lambda_i / (2^{1/2})$ with nonets of scalar σ'_i and pseudoscalar fields π_i which transform according to the $3 \otimes \bar{3} + \bar{3} \otimes 3$ representation of SU(3) \times SU(3)

$$\Omega_1 = \sigma^2 + \pi_i \pi_i - f_\pi^2 \approx 0. \quad (2.5)$$

Now it seems appropriate to comment on the chiral matrix M in (2.2) and the constraint Ω_1 in (2.5). The chiral matrix M breaks the SU(3) \times SU(3) symmetries except the SU_V(3) channel. However, in real physics the SU \times SU(3) symmetries are broken so that our ansatz for M is phenomenologically more realistic than that in the literature [25]. Furthermore the SU_V(3) symmetry is also broken due to the direct SU_V(3) breaking. Even though we have assumed the SU_V(3) symmetry in the Lagrangian (2.4) without an explicit SU_V(3) breaking term for simplicity, the geometrical constraint should break this SU_V(3) symmetry and instead respect the SU(2) flavor symmetry as in (2.5).

By performing the Legendre transformation, one can obtain the canonical Hamiltonian,

$$H_c = \int d^3x \left[\frac{1}{2} (\pi_\sigma^2 + \pi_\pi^a \pi_\pi^a) + \frac{1}{2} ((\partial_i \sigma)^2 + (\partial_i \pi_a)^2) + \frac{1}{2} \mu_0^2 (\sigma^2 + \pi_a \pi_a) + \frac{1}{4} \lambda_0 (\sigma^2 + \pi_a \pi_a)^2 + \bar{\psi} i \gamma^i \partial_i \psi + g_0 \bar{\psi} \frac{1}{\sqrt{2}} (\sigma + i \gamma_5 \pi_a \lambda_a) \psi \right], \quad (2.6)$$

where π_σ and π_π^a are the canonical momenta conjugate to the fields σ and π_a , respectively, given by

$$\pi_\sigma = \dot{\sigma}, \quad \pi_\pi^a = \dot{\pi}_a \quad (2.7)$$

and we have used $\bar{\psi}$ for the canonical momenta conjugate to the fields ψ instead of $\pi_\psi^\dagger = i \psi^\dagger$ for simplicity.

Now we want to construct Noether currents under the SU(3)_L \times SU(3)_R local group transformation. Under an infinitesimal isospin transformation in the SU(3) flavor channel [7]

$$\begin{aligned} \psi &\rightarrow \psi' = (1 - i \epsilon_a \hat{Q}_a) \psi, \\ M &\rightarrow M' = (1 - i \epsilon_a \hat{Q}_a) M (1 + i \epsilon_a \hat{Q}_a), \end{aligned} \quad (2.8)$$

where $\epsilon_a(x)$ are the local angle parameters of the group transformation and $\hat{Q}_a = \lambda_a / 2$ are the SU(3) flavor charge operators given by the generators of the symmetry, the Noether theorem yields the *conserved* flavor octet vector currents (FOVC) for the Lagrangian (2.4)

$$J_V^{\mu a} = \bar{\psi} \gamma^\mu \hat{Q}_a \psi + \frac{i}{2} \text{tr} \left([M, \hat{Q}_a] \partial^\mu M^\dagger + \partial^\mu M [M^\dagger, \hat{Q}_a] \right). \quad (2.9)$$

In addition one can see that the electromagnetic (EM) currents J_{EM}^μ can be easily constructed by replacing the SU(3) flavor charge operators \hat{Q}_a with the EM charge operator $\hat{Q}_{EM} = \hat{Q}_3 + (1/(3^{1/2})) \hat{Q}_8$ in the FOVC (2.9). Moreover one can obtain the charge density ρ as follows:

$$\rho = \psi^\dagger \hat{Q}_{EM} \psi + \left(f_{3ab} + \frac{1}{\sqrt{3}} f_{8ab} \right) \pi_a \pi_\pi^b. \quad (2.10)$$

Now we introduce the chemical potentials $\mu = (\mu_\pi, \mu_K)$ corresponding to the charge densities $\rho = (\rho_\pi, \rho_K)$ in the

pion and kaon flavor channels to yield the Hamiltonian in the kaon condensed matter:

$$H = H_c + \mu_\pi \rho_\pi + \mu_K \rho_K, \quad (2.11)$$

where the charge densities are now explicitly given by

$$\begin{aligned} \rho_\pi &= \psi^\dagger (\hat{Q}_u + \hat{Q}_d) \psi + \pi_1 \pi_\pi^2 - \pi_2 \pi_\pi^1, \\ \rho_K &= \psi^\dagger \hat{Q}_s \psi + \pi_4 \pi_\pi^5 - \pi_5 \pi_\pi^4, \end{aligned} \quad (2.12)$$

with \hat{Q}_q being the q -flavor EM charge operators. Here we have ignored the beta equilibrium for simplicity. Note that by defining the flavor projection operators

$$\hat{P}_{u,d} = \frac{1}{3} \pm \frac{1}{2} \lambda_3 + \frac{1}{2\sqrt{3}} \lambda_8, \quad \hat{P}_s = \frac{1}{3} - \frac{1}{2\sqrt{3}} \lambda_8, \quad (2.13)$$

satisfying $\hat{P}_q^2 = \hat{P}_q$ and $\sum_q \hat{P}_q = 1$, one can easily construct the q -flavor EM charge operators $\hat{Q}_q = \hat{Q}_{\text{EM}} \hat{P}_q = Q_q \hat{P}_q$.

On the other hand, the time evolution of the constraint Ω_1 yields an additional secondary constraint

$$\Omega_2 = \sigma \pi_\sigma + \pi_i \pi_\pi^i \approx 0, \quad (2.14)$$

and Ω_1 and Ω_2 form a second-class constraint algebra:

$$\begin{aligned} \Delta_{kk'}(x, y) &= \{\Omega_k(x), \Omega_{k'}(y)\} \\ &= 2\epsilon^{kk'} (\sigma^2 + \pi_i \pi_i) \delta(x - y), \end{aligned} \quad (2.15)$$

with $\epsilon^{12} = -\epsilon^{21} = 1$.

Using the Dirac brackets [14] defined by

$$\begin{aligned} \{A(x), B(y)\}_{\text{D}} &= \{A(x), B(y)\} \\ &- \int d^3z d^3z' \{A(x), \Omega_k(z)\} \Delta^{kk'} \{\Omega_{k'}(z'), B(y)\}, \end{aligned} \quad (2.16)$$

with $\Delta^{kk'}$ being the inverse of $\Delta_{kk'}$ in (2.15), we obtain the following commutators:

$$\begin{aligned} \{\sigma(x), \sigma(y)\}_{\text{D}} &= \{\pi_\sigma(x), \pi_\sigma(y)\}_{\text{D}} = 0, \\ \{\sigma(x), \pi_\sigma(y)\}_{\text{D}} &= \left(1 - \frac{\sigma^2}{\sigma^2 + \pi_k \pi_k}\right) \delta(x - y), \\ \{\pi_a(x), \pi_b(y)\}_{\text{D}} &= 0, \\ \{\pi_a(x), \pi_\pi^b(y)\}_{\text{D}} &= \left(\delta_{ab} - \frac{\pi_i \pi_j}{\sigma^2 + \pi_k \pi_k} \delta_{ai} \delta_{bj}\right) \delta(x - y), \\ \{\pi_\pi^a(x), \pi_\pi^b(y)\}_{\text{D}} &= \frac{1}{\sigma^2 + \pi_k \pi_k} (\pi_j \pi_\pi^i - \pi_i \pi_\pi^j) \\ &\quad \times \delta_{ai} \delta_{bj} \delta(x - y), \\ \{\psi(x), \psi(y)\}_{\text{D}} &= \{\pi_\psi^\dagger(x), \pi_\psi^\dagger(y)\}_{\text{D}} = 0, \\ \{\psi(x), \pi_\psi^\dagger(y)\}_{\text{D}} &= \delta(x - y). \end{aligned} \quad (2.17)$$

Following the improved Dirac formalism [17, 7] which systematically converts the second-class constraints into first-class ones, we introduce two auxiliary fields (θ, π_θ) with the Poisson brackets

$$\{\theta(x), \pi_\theta(y)\} = \epsilon^{ij} \delta(x - y). \quad (2.18)$$

The first-class constraints $\tilde{\Omega}_i$ are then constructed as

$$\tilde{\Omega}_1 = \Omega_1 + 2\theta, \quad \tilde{\Omega}_2 = \Omega_2 - (\sigma^2 + \pi_k \pi_k) \pi_\theta, \quad (2.19)$$

which satisfy the closed algebra $\{\tilde{\Omega}_i(x), \tilde{\Omega}_j(y)\} = 0$.

3 First-class Hamiltonian

Now, following the improved Dirac formalism [17, 7], we construct the first-class physical fields $\tilde{\mathcal{F}} = (\tilde{\sigma}, \tilde{\pi}_a, \tilde{\psi}, \tilde{\pi}_\sigma, \tilde{\pi}_\pi^a, \tilde{\pi}_\psi)$ corresponding to the original fields $\mathcal{F} = (\sigma, \pi_a, \psi, \pi_\sigma, \pi_\pi^a, \pi_\psi)$. The $\tilde{\mathcal{F}}$'s, which reside in the extended phase space, are obtained as a power series in the auxiliary fields (θ, π_θ) by demanding that they are strongly involutive: $\{\tilde{\Omega}_i, \tilde{\mathcal{F}}\} = 0$. After some lengthy algebra, we obtain the first-class physical fields:

$$\begin{aligned} \tilde{\sigma} &= \sigma \left(\frac{\sigma^2 + \pi_k \pi_k + 2\theta}{\sigma^2 + \pi_k \pi_k} \right)^{1/2}, \\ \tilde{\pi}_i &= \pi_i \left(\frac{\sigma^2 + \pi_k \pi_k + 2\theta}{\sigma^2 + \pi_k \pi_k} \right)^{1/2}, \\ \tilde{\pi}_\sigma &= (\pi_\sigma - \sigma \pi_\theta) \left(\frac{\sigma^2 + \pi_k \pi_k}{\sigma^2 + \pi_k \pi_k + 2\theta} \right)^{1/2}, \\ \tilde{\pi}_\pi^i &= (\pi_\pi^i - \pi_i \pi_\theta) \left(\frac{\sigma^2 + \pi_k \pi_k}{\sigma^2 + \pi_k \pi_k + 2\theta} \right)^{1/2}, \\ \tilde{\pi}_{\bar{a}} &= \pi_{\bar{a}}, \quad \tilde{\pi}_\pi^{\bar{a}} = \pi_\pi^{\bar{a}}, \quad \tilde{\psi} = \psi, \quad \tilde{\pi}_\psi^\dagger = \pi_\psi^\dagger, \end{aligned} \quad (3.1)$$

with the new notation $\bar{a} = 4, 5, 6, 7, 8$.

Since any functional of the first-class fields $\tilde{\mathcal{F}}$ is also first class, we can construct a first-class Hamiltonian in terms of the above first-class physical variables as follows:

$$\begin{aligned} \tilde{H} &= \int d^3x \left[\frac{1}{2} (\tilde{\pi}_\sigma^2 + \tilde{\pi}_\pi^a \tilde{\pi}_\pi^a) + \frac{1}{2} ((\partial_i \tilde{\sigma})^2 + (\partial_i \tilde{\pi}_a)^2) \right. \\ &\quad + \frac{1}{2} \mu_0^2 (\tilde{\sigma}^2 + \tilde{\pi}_a \tilde{\pi}_a) + \frac{1}{4} \lambda_0 (\tilde{\sigma}^2 + \tilde{\pi}_a \tilde{\pi}_a)^2 + \mu_\pi \tilde{\rho}_\pi \\ &\quad \left. + \mu_K \tilde{\rho}_K + \tilde{\psi} i \gamma^i \partial_i \tilde{\psi} + g_0 \tilde{\psi} \frac{1}{\sqrt{2}} (\tilde{\sigma} + i \gamma_5 \tilde{\pi}_a \lambda_a) \tilde{\psi} \right]. \end{aligned} \quad (3.2)$$

We then directly rewrite this Hamiltonian in terms of the original as well as auxiliary fields² to obtain

$$\begin{aligned} \tilde{H} &= \int d^3x \left[\frac{1}{2} ((\pi_\sigma - \sigma \pi_\theta)^2 \right. \\ &\quad + (\pi_\pi^i - \pi_i \pi_\theta)^2) \frac{\sigma^2 + \pi_k \pi_k}{\sigma^2 + \pi_k \pi_k + 2\theta} + \frac{1}{2} \pi_\pi^{\bar{a}} \pi_\pi^{\bar{a}} \\ &\quad + \frac{1}{2} ((\partial_i \sigma)^2 + (\partial_i \pi_k)^2) \\ &\quad + \mu_0^2 (\sigma^2 + \pi_i \pi_i) \frac{\sigma^2 + \pi_k \pi_k + 2\theta}{\sigma^2 + \pi_k \pi_k} + \frac{1}{2} (\partial_i \pi_{\bar{a}})^2 \\ &\quad + \frac{1}{2} \mu_0^2 \pi_{\bar{a}} \pi_{\bar{a}} + \frac{1}{4} \lambda_0 (\sigma^2 + \pi_i \pi_i)^2 \left(\frac{\sigma^2 + \pi_k \pi_k + 2\theta}{\sigma^2 + \pi_k \pi_k} \right)^2 \\ &\quad + \frac{1}{4} \lambda_0 (\pi_{\bar{a}} \pi_{\bar{a}})^2 + \frac{1}{2} \lambda_0 \pi_{\bar{a}} \pi_{\bar{a}} (\sigma^2 + \pi_i \pi_i + 2\theta) + \mu_\pi \rho_\pi \\ &\quad + \mu_K \rho_K + \bar{\psi} i \gamma^i \partial_i \psi \\ &\quad \left. + g_0 \bar{\psi} \frac{1}{\sqrt{2}} (\sigma + i \gamma_5 \pi_i \tau_i) \psi \left(\frac{\sigma^2 + \pi_k \pi_k + 2\theta}{\sigma^2 + \pi_k \pi_k} \right)^{1/2} \right] \end{aligned}$$

² In deriving the first-class Hamiltonian \tilde{H} of (3.3), we have used the conformal map condition, $\sigma \partial_i \sigma + \pi_k \partial_i \pi_k = 0$

$$+ g_0 \bar{\psi} \frac{1}{\sqrt{2}} i \gamma_5 \pi_a \lambda_a \psi \Big], \quad (3.3)$$

where we observe that the forms of the first two terms in this Hamiltonian are exactly the same as those of the O(3) nonlinear sigma model [27].

Now we notice that, even though \tilde{H} is strongly involutive with the first-class constraints $\{\tilde{\Omega}_i, \tilde{H}\} = 0$, it does not naturally generate the first-class Gauss law constraint from the time evolution of the constraint $\tilde{\Omega}_1$. By introducing an additional term proportional to the first-class constraints $\tilde{\Omega}_2$ into \tilde{H} , we then obtain an equivalent first-class Hamiltonian:

$$\tilde{H}' = \tilde{H} + \int d^3x \pi_\theta \tilde{\Omega}_2, \quad (3.4)$$

which naturally generates the Gauss law constraint

$$\{\tilde{\Omega}_1, \tilde{H}'\} = 2\tilde{\Omega}_2, \quad \{\tilde{\Omega}_2, \tilde{H}'\} = 0. \quad (3.5)$$

One notes here that \tilde{H} and \tilde{H}' act in the same way on physical states, which are annihilated by the first-class constraints. Similarly, the equations of motion for observables remain unaffected by the additional term in \tilde{H}' . Furthermore, on the zero locus of the constraints (θ, π_θ) , our first-class system is exactly reduced to the original second-class one.

Next, we consider the Poisson brackets of the fields in the extended phase space $\tilde{\mathcal{F}}$ and identify the Dirac brackets by taking the vanishing limit of auxiliary fields. After some algebraic manipulation starting from (3.1), one can obtain the commutators

$$\begin{aligned} \{\tilde{\sigma}(x), \tilde{\sigma}(y)\} &= \{\tilde{\pi}_\sigma(x), \tilde{\pi}_\sigma(y)\} = 0, \\ \{\tilde{\sigma}(x), \tilde{\pi}_\sigma(y)\} &= \left(1 - \frac{\tilde{\sigma}^2}{\tilde{\sigma}^2 + \tilde{\pi}_k \tilde{\pi}_k}\right) \delta(x-y), \\ \{\tilde{\pi}_a(x), \tilde{\pi}_b(y)\} &= 0, \\ \{\tilde{\pi}_a(x), \tilde{\pi}_\pi^b(y)\} &= \left(\delta_{ab} - \frac{\tilde{\pi}_i \tilde{\pi}_j}{\tilde{\sigma}^2 + \tilde{\pi}_k \tilde{\pi}_k} \delta_{ai} \delta_{bj}\right) \delta(x-y), \\ \{\tilde{\pi}_\pi^a(x), \tilde{\pi}_\pi^b(y)\} &= \frac{1}{\tilde{\sigma}^2 + \tilde{\pi}_k \tilde{\pi}_k} (\tilde{\pi}_j \tilde{\pi}_\pi^i - \tilde{\pi}_i \tilde{\pi}_\pi^j) \\ &\quad \times \delta_{ai} \delta_{bj} \delta(x-y), \\ \{\tilde{\psi}(x), \tilde{\psi}(y)\} &= \{\tilde{\pi}_\psi^\dagger(x), \tilde{\pi}_\psi^\dagger(y)\} = 0, \\ \{\tilde{\psi}(x), \tilde{\pi}_\psi^\dagger(y)\} &= \delta(x-y). \end{aligned} \quad (3.6)$$

One notes here that on the zero locus of the constraints (θ, π_θ) , the above Poisson brackets in the extended phase space exactly reproduce the corresponding Dirac brackets (2.17). It is also noteworthy that the Poisson brackets of the fields $\tilde{\mathcal{F}}$ in (3.6) have exactly the same form as those of the Dirac brackets of the field \mathcal{F} , yielding $\{\tilde{A}, \tilde{B}\} = \{A, B\}_{\text{D}}|_{A \rightarrow \tilde{A}, B \rightarrow \tilde{B}}$. On the other hand, this kind of situation occurs again when one considers the first-class constraints (2.19). More precisely, these first-class constraints in the extended phase space can be rewritten as

$$\tilde{\Omega}_1 = \tilde{\sigma}^2 + \tilde{\pi}_i \tilde{\pi}_i - f_\pi^2, \quad \tilde{\Omega}_2 = \tilde{\sigma} \tilde{\pi}_\sigma + \tilde{\pi}_i \tilde{\pi}_\pi^i, \quad (3.7)$$

which are form-invariant with respect to the second-class constraints (2.5) and (2.14).

4 BRST symmetries

In this section, we will obtain the BRST invariant Lagrangian in the framework of the BFV formalism [23] which is applicable to theories with the first-class constraints by introducing two canonical sets of ghosts and anti-ghosts together with auxiliary fields $(\mathcal{C}^i, \bar{\mathcal{P}}_i)$, $(\mathcal{P}^i, \bar{\mathcal{C}}_i)$, (N^i, B_i) ($i = 1, 2$), which satisfy the super-Poisson algebra

$$\begin{aligned} \{\mathcal{C}^i(x), \bar{\mathcal{P}}_j(y)\} &= \{\mathcal{P}^i(x), \bar{\mathcal{C}}_j(y)\} = \{N^i(x), B_j(y)\} \\ &= \delta_j^i \delta(x-y). \end{aligned} \quad (4.1)$$

Here the super-Poisson bracket is defined by

$$\{A, B\} = \frac{\delta A}{\delta q} \Big|_r \frac{\delta B}{\delta p} \Big|_l - (-1)^{\eta_A \eta_B} \frac{\delta B}{\delta q} \Big|_r \frac{\delta A}{\delta p} \Big|_l, \quad (4.2)$$

where η_A denotes the number of fermions, called the ghost number, in A and the subscript r and l right and left derivatives.

In this phenomenological SU(3) linear sigma model, the nilpotent BRST charge Q , the fermionic gauge fixing function Ψ and the BRST invariant minimal Hamiltonian H_m are given by

$$\begin{aligned} Q &= \int d^3x (\mathcal{C}^i \tilde{\Omega}_i + \mathcal{P}^i B_i), \\ \Psi &= \int d^3x (\bar{\mathcal{C}}_i \chi^i + \bar{\mathcal{P}}_i N^i), \\ H_m &= \tilde{H}' - \int d^3x 2\mathcal{C}^1 \bar{\mathcal{P}}_2, \end{aligned} \quad (4.3)$$

which satisfy the relations $\{Q, H_m\} = 0$, $Q^2 = \{Q, Q\} = 0$, $\{\Psi, Q\}, Q\} = 0$. The effective quantum Lagrangian is then described by

$$\begin{aligned} L_{\text{eff}} &= \int d^3x (\pi_\sigma \dot{\sigma} + \pi_\pi^a \dot{\pi}_a + \pi_\psi^\dagger \dot{\psi} + \pi_\theta \dot{\theta} \\ &\quad + B_2 \dot{N}^2 + \bar{\mathcal{P}}_i \dot{\mathcal{C}}^i + \bar{\mathcal{C}}_2 \dot{\mathcal{P}}^2) - H_{\text{tot}}, \end{aligned} \quad (4.4)$$

with $H_{\text{tot}} = H_m - \{Q, \Psi\}$. Here the $B_1 \dot{N}^1 + \bar{\mathcal{C}}_1 \dot{\mathcal{P}}^1 = \{Q, \bar{\mathcal{C}}_1 \dot{N}^1\}$ terms are suppressed by replacing χ^1 with $\chi^1 + \dot{N}^1$.

Now we choose the unitary gauge

$$\chi^1 = \Omega_1, \quad \chi^2 = \Omega_2, \quad (4.5)$$

and perform the path integration over the fields B_1 , N^1 , $\bar{\mathcal{C}}_1$, \mathcal{P}^1 , $\bar{\mathcal{P}}_1$ and \mathcal{C}^1 , by using the equations of motion, to yield the effective Lagrangian of the form

$$\begin{aligned} L_{\text{eff}} &= \int d^3x \left[\pi_\sigma \dot{\sigma} + \pi_\pi^a \dot{\pi}_a + \pi_\psi^\dagger \dot{\psi} + \pi_\theta \dot{\theta} \right. \\ &\quad \left. + B_2 \dot{N}^2 + \bar{\mathcal{P}}_2 \dot{\mathcal{C}}^2 + \bar{\mathcal{C}}_2 \dot{\mathcal{P}}^2 \right] \end{aligned}$$

$$\begin{aligned}
& -\frac{R}{2} \left((\pi_\sigma - \sigma\pi_\theta)^2 + (\pi_\pi^i - \pi_i\pi_\theta)^2 \right) - \frac{1}{2} \pi_\pi^{\bar{a}} \pi_\pi^{\bar{a}} \\
& - \frac{1}{2R} \left((\partial_i\sigma)^2 + (\partial_i\pi_k)^2 \right) - \frac{1}{2} (\partial_i\pi_{\bar{a}})^2 \\
& - \frac{1}{2} \mu_0^2 \left(\frac{1}{R} (\sigma^2 + \pi_i\pi_i) + \pi_{\bar{a}}\pi_{\bar{a}} \right) \\
& - \frac{1}{4} \lambda_0 \left(\frac{1}{R} (\sigma^2 + \pi_i\pi_i) + \pi_{\bar{a}}\pi_{\bar{a}} \right)^2 - \bar{\psi} i\gamma^i \partial_i \psi \\
& - g_0 \bar{\psi} \frac{1}{\sqrt{2R}} (\sigma + i\gamma_5 \pi_i \tau_i) \psi - g_0 \bar{\psi} \frac{1}{\sqrt{2}} i\gamma_5 \pi_{\bar{a}} \lambda_{\bar{a}} \psi \\
& + (\sigma\pi_\sigma + \pi_i\pi_\pi^i - (\sigma^2 + \pi_k\pi_k)\pi_\theta) (-\pi_\theta + N) \\
& - 2(\sigma^2 + \pi_k\pi_k)\pi_\theta \bar{\mathcal{C}} \\
& + (\sigma\pi_\sigma + \pi_i\pi_\pi^i)B + \bar{\mathcal{P}}\mathcal{P} - \mu_\pi\rho_\pi - \mu_K\rho_K \Big], \quad (4.6)
\end{aligned}$$

with the redefinitions: $N \equiv N^2$, $B \equiv B_2$, $\bar{\mathcal{C}} \equiv \bar{\mathcal{C}}_2$, $\mathcal{C} \equiv \mathcal{C}^2$, $\bar{\mathcal{P}} \equiv \bar{\mathcal{P}}_2$, $\mathcal{P} \equiv \mathcal{P}_2$ and $R = (\sigma^2 + \pi_i\pi_i)/(\sigma^2 + \pi_i\pi_i + 2\theta)$.

Next, using the variations with respect to π_σ , $\pi_\pi^{\bar{a}}$, π_θ , \mathcal{P} and $\bar{\mathcal{P}}$, one obtains the relations

$$\begin{aligned}
\dot{\sigma} &= (\pi_\sigma - \sigma\pi_\theta)R + \sigma(\pi_\theta - N - B), \\
\dot{\pi}_i &= (\pi_\pi^i - \pi_i\pi_\theta)R + \pi_i(\pi_\theta - N - B) + \mu_\pi(\pi_1\delta_i^2 - \pi_2\delta_i^1), \\
\dot{\pi}_{\bar{a}} &= \pi_{\bar{a}} + \mu_K(\pi_4\delta_{\bar{a}}^5 - \pi_5\delta_{\bar{a}}^4), \\
\dot{\theta} &= -\sigma(\pi_\sigma - \sigma\pi_\theta)R - \pi_i(\pi_\pi^i - \pi_i\pi_\theta)R \\
&+ (\sigma^2 + \pi_i\pi_i)(-2\pi_\theta + N + 2\bar{\mathcal{C}}) + \sigma\pi_\sigma + \pi_i\pi_\pi^i, \\
\mathcal{P} &= -\dot{\mathcal{C}}, \quad \bar{\mathcal{P}} = \dot{\bar{\mathcal{C}}}, \quad (4.7)
\end{aligned}$$

to yield the effective Lagrangian

$$\begin{aligned}
L_{\text{eff}} &= \int d^3x \left[\frac{1}{2R} (\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi_i\partial^\mu\pi_i) + \frac{1}{2} \partial_\mu\pi_{\bar{a}}\partial^\mu\pi_{\bar{a}} \right. \\
&- \frac{1}{2} \mu_0^2 \left(\frac{1}{R} (\sigma^2 + \pi_i\pi_i) + \pi_{\bar{a}}\pi_{\bar{a}} \right) \\
&- \frac{1}{4} \lambda_0 \left(\frac{1}{R} (\sigma^2 + \pi_i\pi_i) + \pi_{\bar{a}}\pi_{\bar{a}} \right)^2 + \bar{\psi} i\gamma^\mu \partial_\mu \psi \\
&- g_0 \bar{\psi} \frac{1}{\sqrt{2R}} (\sigma + i\gamma_5 \pi_i \tau_i) \psi - g_0 \bar{\psi} \frac{1}{\sqrt{2}} i\gamma_5 \pi_{\bar{a}} \lambda_{\bar{a}} \psi \\
&- \frac{1}{2R} (\sigma^2 + \pi_i\pi_i) \left(\frac{\dot{\theta}}{\sigma^2 + \pi_k\pi_k} + (B + 2\bar{\mathcal{C}})R \right)^2 \\
&+ \frac{1}{R} (B + N) \left(-\dot{\theta} + (\sigma^2 + \pi_i\pi_i) \right. \\
&\times \left. \left(\frac{\dot{\theta}}{\sigma^2 + \pi_k\pi_k} + (B + 2\bar{\mathcal{C}})R \right) \right) \\
&\left. + B\dot{N} - \bar{\mathcal{C}}\partial_0^2\mathcal{C} - \mu_\pi\rho_\pi - \mu_K\rho_K \right]. \quad (4.8)
\end{aligned}$$

Finally, with the identification $N = -B + \dot{\theta}/(\sigma^2 + \pi_i\pi_i)$, one can arrive at the BRST invariant Lagrangian

$$L_{\text{eff}} = \int d^3x \left[\frac{1}{2} (\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi_i\partial^\mu\pi_i) \frac{\sigma^2 + \pi_k\pi_k + 2\theta}{\sigma^2 + \pi_k\pi_k} \right.$$

$$\begin{aligned}
& + \frac{1}{2} \partial_\mu\pi_{\bar{a}}\partial^\mu\pi_{\bar{a}} - \frac{1}{2} \mu_0^2 (\sigma^2 + \pi_a\pi_a + 2\theta) \\
& - \frac{1}{4} \lambda_0 (\sigma^2 + \pi_a\pi_a + 2\theta)^2 + \bar{\psi} i\gamma^\mu \partial_\mu \psi \\
& - g_0 \bar{\psi} \frac{1}{\sqrt{2}} (\sigma + i\gamma_5 \pi_i \tau_i) \psi \left(\frac{\sigma^2 + \pi_k\pi_k + 2\theta}{\sigma^2 + \pi_k\pi_k} \right)^{1/2} \\
& - g_0 \bar{\psi} \frac{1}{\sqrt{2}} i\gamma_5 \pi_{\bar{a}} \lambda_{\bar{a}} \psi - \frac{1}{2} \frac{\sigma^2 + \pi_k\pi_k + 2\theta}{(\sigma^2 + \pi_k\pi_k)^2} \dot{\theta}^2 \\
& - \frac{\dot{B}\dot{\theta}}{\sigma^2 + \pi_i\pi_i} - \frac{1}{2} \frac{(\sigma^2 + \pi_k\pi_k)^2}{\sigma^2 + \pi_k\pi_k + 2\theta} (B + 2\bar{\mathcal{C}})^2 \\
& \left. - \bar{\mathcal{C}}\partial_0^2\mathcal{C} - \mu_\pi\rho_\pi - \mu_K\rho_K \right], \quad (4.9)
\end{aligned}$$

which is invariant under the BRST transformation

$$\begin{aligned}
\delta_B\sigma &= \lambda\sigma\mathcal{C}, \quad \delta_B\pi_i = \lambda\pi_i\mathcal{C}, \quad \delta_B\pi_{\bar{a}} = 0, \quad \delta_B\psi = \lambda\psi\mathcal{C}, \\
\delta_B\theta &= -\lambda(\sigma^2 + \pi_i\pi_i)\mathcal{C}, \quad \delta_B\bar{\mathcal{C}} = -\lambda B, \quad \delta_B\mathcal{C} = \delta_B B = 0. \quad (4.10)
\end{aligned}$$

5 Phenomenology and discussion

In this section, to discuss the phenomenological aspects, we exploit the first-class constraints $\tilde{\Omega}_i = 0$ in (2.19) for the Hamiltonian (3.4) to obtain the relation

$$\begin{aligned}
& \frac{1}{2} \left((\pi_\sigma - \sigma\pi_\theta)^2 + (\pi_\pi^i - \pi_i\pi_\theta)^2 \right) \frac{\sigma^2 + \pi_k\pi_k}{\sigma^2 + \pi_k\pi_k + 2\theta} \\
& = \frac{1}{2f_\pi^2} \left((\sigma^2 + \pi_i\pi_i) (\pi_\sigma^2 + \pi_\pi^i\pi_\pi^i) - (\sigma\pi_\sigma + \pi_i\pi_\pi^i)^2 \right). \quad (5.1)
\end{aligned}$$

Following the symmetrization procedure, we then obtain a Hamiltonian of the form

$$\begin{aligned}
\tilde{H} &= \int d^3x \left[\frac{1}{2} (\pi_\sigma^2 + \pi_\pi^i\pi_\pi^i + 1) + \frac{1}{2} \pi_\pi^{\bar{a}}\pi_\pi^{\bar{a}} \right. \\
&+ \frac{1}{2} ((\partial_i\sigma)^2 + (\partial_i\pi_k)^2) + \frac{1}{2} (\partial_i\pi_{\bar{a}})^2 \\
&+ \frac{1}{2} \mu_0^2 (\sigma^2 + \pi_i\pi_i) + \frac{1}{2} \mu_0^2 \pi_{\bar{a}}\pi_{\bar{a}} + \frac{1}{4} \lambda_0 (\sigma^2 + \pi_i\pi_i)^2 \\
&+ \frac{1}{4} \lambda_0 (\pi_{\bar{a}}\pi_{\bar{a}})^2 + \frac{1}{2} \lambda_0 \pi_{\bar{a}}\pi_{\bar{a}} (\sigma^2 + \pi_i\pi_i) \\
&+ \mu_\pi\rho_\pi + \mu_K\rho_K + \bar{\psi} i\gamma^i \partial_i \psi \\
&\left. + g_0 \bar{\psi} \frac{1}{\sqrt{2}} (\sigma + i\gamma_5 \pi_i \tau_i) \psi + g_0 \bar{\psi} \frac{1}{\sqrt{2}} i\gamma_5 \pi_{\bar{a}} \lambda_{\bar{a}} \psi \right]. \quad (5.2)
\end{aligned}$$

Here one notes that the Weyl ordering correction 1/2 in the first line of (5.2) originates from the improved Dirac scheme associated with the geometric constraint (2.5). Moreover, this correction comes only with the kinetic terms, without any dependence on the potential terms.

Now we define mean fields for the Goldstone boson fields as³

$$\begin{aligned}
\langle \sigma \rangle &= \sigma, \quad \langle \pi^\pm \rangle = \pi^\pm, \quad \langle K^\pm \rangle = K^\pm, \\
\langle \pi_\sigma \rangle &= p_\sigma, \quad \langle \pi_{\pi^\pm} \rangle = p_{\pi^\pm}, \quad \langle \pi_{K^\pm} \rangle = p_{K^\pm}, \\
\text{others} &= 0, \quad (5.3)
\end{aligned}$$

³ Here we ignore the eta fields for simplicity

where

$$\begin{aligned}\pi^\pm &= \frac{1}{\sqrt{2}}(\pi_1 \mp i\pi_2), \quad \pi^0 = \pi_3, \quad K^\pm = \frac{1}{\sqrt{2}}(\pi_4 \mp i\pi_5), \\ K^0 &= \frac{1}{\sqrt{2}}(\pi_6 - i\pi_7), \quad \bar{K}^0 = \frac{1}{\sqrt{2}}(\pi_6 + i\pi_7),\end{aligned}\quad (5.4)$$

and we have similar relations for the momenta fields. We then finally arrive at the energy spectrum of the form $\langle \tilde{H} \rangle = \int d^3x \varepsilon$ with

$$\begin{aligned}\varepsilon &= \frac{1}{2}p_\sigma^2 + p_{\pi^+}p_{\pi^-} + p_{K^+}p_{K^-} + \frac{1}{2}(\partial_i\sigma)^2 + \partial_i\pi^+\partial_i\pi^- \\ &+ \partial_iK^+\partial_iK^- + \frac{1}{2}\mu_0^2(\sigma^2 + 2\pi^+\pi^- + 2K^+K^-) \\ &+ \frac{1}{4}\lambda_0(\sigma^2 + 2\pi^+\pi^- + 2K^+K^-)^2 + \bar{\psi}i\gamma^i\partial_i\psi \\ &+ g_0\bar{\psi}\left(\frac{1}{\sqrt{2}}\sigma + i\gamma_5(\tau^+\pi^- + \tau^-\pi^+ + \lambda^+K^- + \lambda^-K^+)\right)\psi \\ &+ i\mu_\pi(\pi^-p_{\pi^+} - \pi^+p_{\pi^-}) + i\mu_K(K^-p_{K^+} - K^+p_{K^-}) \\ &+ \psi^\dagger\left(\mu_\pi(\hat{Q}_u + \hat{Q}_d) + \mu_K\hat{Q}_s\right)\psi + \frac{1}{2},\end{aligned}\quad (5.5)$$

where

$$\tau^\pm = \frac{1}{2}(\tau_1 \mp i\tau_2), \quad \tau^0 = \tau_3, \quad \lambda^\pm = \frac{1}{2}(\lambda_4 \mp i\lambda_5). \quad (5.6)$$

Using the variations with respect to p_σ , p_{π^\pm} and p_{K^\pm} , we obtain the relations

$$p_\sigma = 0, \quad p_{\pi^\pm} = \pm i\mu_\pi\pi^\pm, \quad p_{K^\pm} = \pm i\mu_KK^\pm \quad (5.7)$$

to yield

$$\dot{\sigma} = 0, \quad \dot{\pi}^\pm = \pm i\mu_\pi\pi^\pm, \quad \dot{K}^\pm = \pm i\mu_KK^\pm. \quad (5.8)$$

Substituting (5.7) into the energy spectrum (5.5) and ignoring the irrelevant term $(\partial_i\sigma)^2$, we finally arrive at

$$\begin{aligned}\varepsilon &= \partial_i\pi^+\partial_i\pi^- + \partial_iK^+\partial_iK^- + \frac{1}{2}\mu_0^2\sigma^2 - (\mu_\pi^2 - \mu_0^2)\pi^+\pi^- \\ &- (\mu_K^2 - \mu_0^2)K^+K^- \\ &+ \frac{1}{4}\lambda_0(\sigma^2 + 2\pi^+\pi^- + 2K^+K^-)^2 + \bar{\psi}i\gamma^i\partial_i\psi \\ &+ g_0\bar{\psi}\left(\frac{1}{\sqrt{2}}\sigma + i\gamma_5(\tau^+\pi^- + \tau^-\pi^+ + \lambda^+K^- + \lambda^-K^+)\right)\psi \\ &+ \psi^\dagger\left(\mu_\pi(\hat{Q}_u + \hat{Q}_d) + \mu_K\hat{Q}_s\right)\psi + \frac{1}{2},\end{aligned}\quad (5.9)$$

which still respects the SU(3) flavor symmetry.

6 Conclusion

In summary, we constructed the SU(3) linear sigma model by introducing a novel matrix for the Goldstone bosons which satisfy geometrical second-class constraints. Following the improved Dirac method, we also constructed first-class physical fields and, in terms of them, we directly

obtained a first-class Hamiltonian which is consistent with the Hamiltonian with the original fields and auxiliary fields. The Poisson brackets of the first-class physical fields are also constructed and these Poisson brackets are shown to reproduce the corresponding Dirac brackets in the limit of vanishing auxiliary fields. Exploiting the first-class Hamiltonian, we constructed the BRST invariant effective Lagrangian and its corresponding BRST transformation rules in this phenomenological SU(3) linear sigma model. Finally, defining the mean fields for the Goldstone bosons fields, we obtained the energy spectrum of the corresponding pion and kaon condensates. However this energy spectrum still possesses the SU(3) flavor symmetry. Through further investigation, it will be interesting to study the flavor symmetry breaking effects in the framework of this SU(3) linear sigma model to predict the more realistic kaon condensation.

Acknowledgements. STH would like to thank the hospitality of the Institute of Physics and Applied Physics, Yonsei University where a part of this work has been carried out. We would like to thank S. Ando, K. Kubodera, F. Myhrer, Y. Oh, Y.J. Park and M. Rho for helpful discussions. STH acknowledges financial support in part from the Korea Research Foundation, Grant No. KRF-2001-DP0083. SHL acknowledges financial support from KOSEF under grant No. 1999-2-111-005-5, and the Korean Ministry of Education under Grant No. 2000-2-0689.

References

1. R. Hasty et al., *Science* **290**, 2117 (2000); D.T. Spayde et al., *Phys. Rev. Lett.* **84**, 1106 (2000); B. Mueller et al., *Phys. Rev. Lett.* **78**, 3824 (1997)
2. R.D. McKeown, *Phys. Lett. B* **219**, 140 (1989); E.J. Beise, R.D. McKeown, *Comm. Nucl. Part. Phys.* **20**, 105 (1991)
3. S.T. Hong, B.Y. Park, D.P. Min, *Phys. Lett. B* **414**, 229 (1997); S.T. Hong, B.Y. Park, *Nucl. Phys. A* **561**, 525 (1993)
4. G.E. Brown, M. Rho, *Phys. Lett. B* **82**, 177 (1979)
5. H.C. Kim, M. Praszalowicz, M.V. Polyakov, K. Goeke, *Phys. Rev. D* **58**, 114027 (1998)
6. U.G. Meissner, *Nucl. Phys. A* **666–667**, 51 (2000); C.M. Maekawa, U. van Kolck, *Phys. Lett. B* **478**, 73 (2000); C.M. Maekawa, J.S. Veiga, U. van Kolck, *Phys. Lett. B* **488**, 167 (2000); S.L. Zhu, S.J. Puglia, B.R. Holstein, M.J. Ramsey-Musolf, *Phys. Rev. D* **62**, 033008 (2000); W.C. Haxton, C.P. Liu, M.J. Ramsey-Musolf, *Phys. Rev. Lett.* **86**, 5247 (2001); D.H. Beck, R.D. McKeown, *hep-ph/0102334*
7. S.T. Hong, Y.J. Park, *Phys. Rep.* **358**, 143 (2002) and references therein
8. D.B. Kaplan, A.E. Nelson, *Phys. Lett. B* **175**, 57 (1986)
9. G.E. Brown, K. Kubodera, M. Rho, *Phys. Lett. B* **192**, 273 (1987); H.D. Politzer, M.B. Wise, *Phys. Lett. B* **273**, 156 (1991); G.E. Brown, K. Kubodera, M. Rho, V. Thorsson, *Phys. Lett. B* **291**, 355 (1992)
10. G.E. Brown, M. Rho, C. Song, *Nucl. Phys. A* **690**, 184 (2001); J.A. Pons, A.W. Steiner, M. Prakash, J.M. Lattimer, *Phys. Rev. Lett.* **86**, 5223 (2001); M. Yasuhira,

- T. Tatsumi, Nucl. Phys. A **690**, 769 (2001); T. Norsen, S. Reddy, Phys. Rev. C **63**, 065804 (2001); T. Schaefer, D.T. Son, M.A. Stephanov, D. Toublan, J.J.M. Verbaarschot, Phys. Lett. B **522**, 67 (2001); V.A. Miransky, I.A. Shovkovy, Phys. Rev. Lett. **88**, 111601 (2002); P.F. Bedaque, Phys. Lett. B **524**, 137 (2002); A. Ramos, J. Schaffner-Bielich, J. Wambach, Lect. Notes Phys. **578**, 175 (2001)
11. T. Schafer, Phys. Rev. Lett. **26**, 5531 (2000); M. Alford, K. Rajagopal, S. Reddy, F. Wilczek, Phys. Rev. D **64**, 074017 (2001); T. Schaefer, E. Shuryak, Lect. Notes Phys. **578**, 203 (2001)
12. G.E. Brown, H.A. Bethe, Astrophys. J. **423**, 659 (1994)
13. H. Yabu, S. Nakamura, F. Myhrer, K. Kubodera, Phys. Lett. B **315**, 17 (1993)
14. P.A.M. Dirac, Lectures in quantum mechanics (Yeshiva University, New York 1964)
15. S.H. Lee, I. Zahed, Phys. Rev. D **37**, 1963 (1988)
16. S.H. Lee, I. Zahed, Phys. Lett. B **215**, 583 (1988)
17. I.A. Batalin, E.S. Fradkin, Phys. Lett. B **180**, 157 (1986); Nucl. Phys. B **279**, 514 (1987); I.A. Batalin, I.V. Tyutin, Int. J. Mod. Phys. A **6**, 3255 (1991)
18. S.T. Hong, Y.W. Kim, Y.J. Park, Phys. Rev. D **59**, 114026 (1999)
19. S.T. Hong, Y.W. Kim, Y.J. Park, Mod. Phys. Lett. A **15**, 55 (2000)
20. S.T. Hong, Y.J. Park, Phys. Rev. D **63**, 054018 (2001)
21. D.K. Hong, S.T. Hong, Y.J. Park, Phys. Lett. B **499**, 125 (2001)
22. C. Becchi, A. Rouet, R. Stora, Ann. Phys. **98**, 287 (1976); I.V. Tyutin, Lebedev Preprint 39 (1975)
23. E.S. Fradkin, G.A. Vilkovisky, Phys. Lett. B **55**, 224 (1975); M. Henneaux, Phys. Rev. C **126**, 1 (1985); T. Fujiwara, Y. Igarashi, J. Kubo, Nucl. Phys. B **341**, 695 (1990); Y.W. Kim, S.K. Kim, W. T. Kim, Y.J. Park, K.Y. Kim, Y. Kim, Phys. Rev. D **46**, 4574 (1992); C. Bizdadea, S.O. Saliu, Nucl. Phys. B **456**, 473 (1995); S. Hamamoto, Prog. Theor. Phys. **95**, 441 (1996)
24. H.J. Lee, J.H. Yee, Phys. Rev. D **47**, 4608 (1993); Phys. Lett. B **320**, 52 (1994)
25. M. Levy, Nuo. Cim. A **52**, 23 (1967); S. Gasiorowicz, D.A. Geffen, Rev. Mod. Phys. **41**, 531 (1969); H. Pagels, Phys. Rep. **16**, 219 (1975); D. Black, A.H. Fariborz, S. Moussa, S. Nasri, J. Schechter, Phys. Rev. D **64**, 014031 (2001)
26. S.T. Hong, D.P. Min, J. Korean Phys. Soc. **30**, 516 (1997)
27. S.T. Hong, W.T. Kim, Y.J. Park, Phys. Rev. D **60**, 125005 (1999)